

Mathematics II

(English course)

Second semester, 2012/2013

Exercises (8)

1. Compute the following integrals.

- (a) $\iint_A (y-1)x^2 dx dy$,
 $A = \{(x, y) : 0 \leq x \leq 1 \wedge 0 \leq y \leq 2 \wedge y \leq 2x\}$.
- (b) $\iint_A \frac{y}{1+x^2} dx dy$, $A = \{(x, y) : x^2 - x \leq y \leq x\}$.
- (c) $\iint_A xy + x^2 dx dy$,
 $A = \{(x, y) : x - 2 \leq y \leq 1 + x \wedge 1 - \frac{x}{2} \leq y \leq \frac{5-x}{2}\}$.
- (d) $\int_0^1 \int_0^1 f(x, y) dx dy$, $f(x, y) = \begin{cases} x - y, & \text{for } y \geq x, \\ (x - y)^2 & \text{for } y \leq x. \end{cases}$
- (e) $\iint_A y dx dy$, $A = \{(x, y) : x^2 - 2\pi x \leq y \leq \sin x\}$.
- (f) $\iint_A xy dx dy$, $A = \{(x, y) : y^2 \leq x \leq 1 + y^2 \wedge x \leq 2 - y^2\}$.
- (g) $\iint_A (x+1)e^{xy} dx dy$,
 $A = \{(x, y) : 1 \leq x + 2y \leq 2 \wedge -1 \leq 2x + y \leq 1\}$.
- (h) $\iiint_A x + 2y + 3z dx dy dz$,
 $A = \{(x, y, z) : x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1\}$.
- (i) $\iiint_A xy^2 z^3 dx dy dz$,
 $A = \{(x, y, z) : x \geq 0, y \geq 0, 0 \leq z \leq 1, x + y + z \leq 2\}$.

2. Compute the following integrals.

- (a) $\iint_A \frac{1}{1+x^2+y^2} dx dy$, $A = \{(x, y) : x^2 + y^2 \leq 4\}$.
- (b) $\iint_A x^2 + y^2 dx dy$, $A = \{(x, y) : x^2 + y^2 \leq 2x\}$.
- (c) $\iiint_A z dx dy dz$, $A = \{(x, y, z) : x^2 + y^2 \leq z \leq 1\}$.
- (d) $\iiint_A z dx dy dz$,
 $A = \{(x, y, z) : \sqrt{x^2 + y^2} \leq z \wedge x^2 + y^2 + z^2 \leq 1\}$.

3. Compute the following integrals.

- (a) $\iint_A e^{-xy} dx dy$, $A = \{(x, y) : 0 \leq y \leq 2 \wedge x > 0\}$.
- (b) $\iint_A \frac{1}{x+y} dx dy$, $A = \{(x, y) : x - y \geq 1 \wedge 0 \leq xy \leq 1\}$.

(c) $\iint_A \ln(x+y) dx dy$, $A = \{(x, y) : x \geq 0, y \geq 0, x+y \leq 1\}$.

4. Compute the area of the following sets.

(a) The region bounded by the lines $y = x$, $x = 2y$, $x + y = 1$, and $x + 3y = 1$.

(b) The region bounded by the lines $y^2 = 10x + 25$, and $y^2 + 6x = 9$.

(c) The set $A = \{(x, y) : x + y \geq 1 \wedge x^2 + y^2 \leq 1\}$.

(d) The set $A = \left\{ (x, y) : \frac{x^3}{1+x^2} \leq y \leq x \right\}$.